

CHAPTER 4 POWER SYSTEM STABILITY

Power system stability refers to the ability of synchronous machines to move from one steady-state operating point following a disturbance to another steady-state operating point, without losing synchronism.

Three types of power system stability:

1. **Steady-state stability.** Involves slow or gradual changes in operating points. Steady-state stability studies, which are usually performed with a power-flow computer program, ensure that phase angles across transmission lines are not too large, that bus voltages are close to nominal values, and that generators, transmission lines, transformers, and other equipment are not overloaded.
2. **Transient stability.** Involves major disturbances such as loss of generation. Following a disturbance, synchronous machine frequencies undergo transient deviations from synchronous frequency (60 Hz), and machine power angles change. The objective of a transient stability study is to determine whether or not the machines will return to synchronous frequency with new steady-state power angles. Changes in power flows and bus voltages are also of concern. In many cases, transient stability is determined during the first swing of machine power angles following a disturbance. Where multisiwings lasting several seconds are of concern, models of turbine-governors and excitation systems as well as more detailed machine models can be employed to obtain accurate transient stability results over the longer time period.
3. **Dynamic stability.** Involves an even longer time period, typically several minutes. It is possible for controls to affect dynamic stability even though transient stability is maintained. The action of turbine-governors, excitation systems, tap-changing transformers, and controls from a power system dispatch center can interact to stabilize or destabilize a power system several minutes after a disturbance has occurred.

MEDIUM AND SHORT LINE APPROXIMATIONS

ABCD Parameters

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad (5.1.3)$$

FIGURE 5.2

Short transmission line

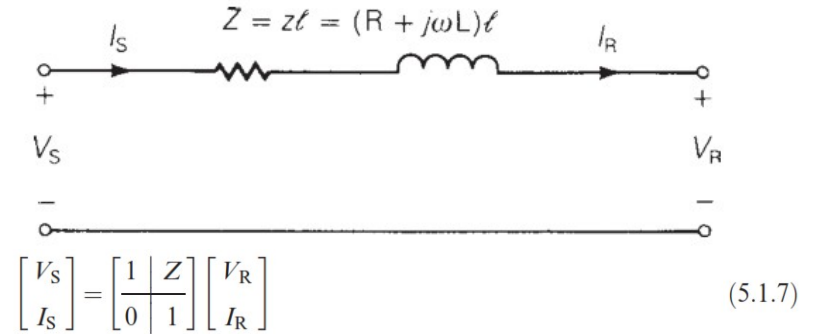
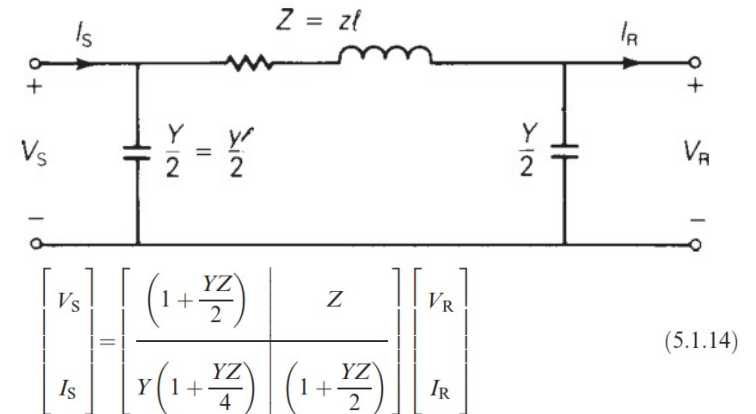


FIGURE 5.3

Medium-length transmission line—nominal π circuit



Where:

$$z = R + j\omega L \quad \Omega/\text{m}, \text{ series impedance per unit length}$$

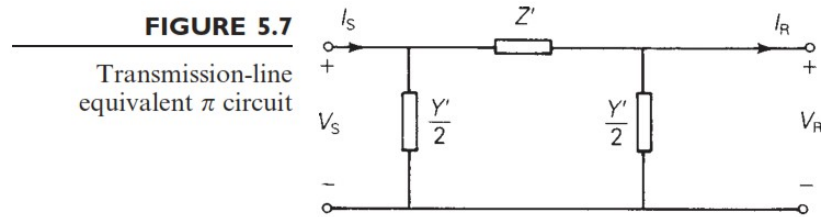
$$y = G + j\omega C \quad \text{S/m}, \text{ shunt admittance per unit length}$$

$$Z = zl \quad \Omega, \text{ total series impedance}$$

$$Y = yl \quad \text{S}, \text{ total shunt admittance}$$

$$l = \text{line length} \quad \text{m}$$

EQUIVALENT π CIRCUIT OF A LONG TRANSMISSION LINE



$$Z' = Z_c \sinh(\gamma \ell) = Z F_1 = Z \frac{\sinh(\gamma \ell)}{\gamma \ell}$$

$$\frac{Y'}{2} = \frac{\tanh(\gamma \ell / 2)}{Z_c} = \frac{Y}{2} F_2 = \frac{Y \tanh(\gamma \ell / 2)}{(\gamma \ell / 2)}$$

Propagation constant:

$$\gamma = \sqrt{zy} \quad \text{m}^{-1} \quad (5.2.12)$$

Characteristic impedance:

$$Z_c = \sqrt{\frac{z}{y}} \quad \Omega \quad (5.2.16)$$

LOSSLESS LINES

For a lossless line, $R = G = 0$, and

$$z = j\omega L \quad \Omega/\text{m} \quad (5.4.1)$$

$$y = j\omega C \quad \text{S/m} \quad (5.4.2)$$

Surge impedance = characteristic impedance for a lossless line:

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \quad \Omega \quad (5.4.3)$$

Propagation constant:

$$\gamma = \sqrt{zy} = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \quad \text{m}^{-1} \quad (5.4.4)$$

where

$$\beta = \omega\sqrt{LC} \quad \text{m}^{-1} \quad (5.4.5)$$

Equivalent π circuit of a long transmission line

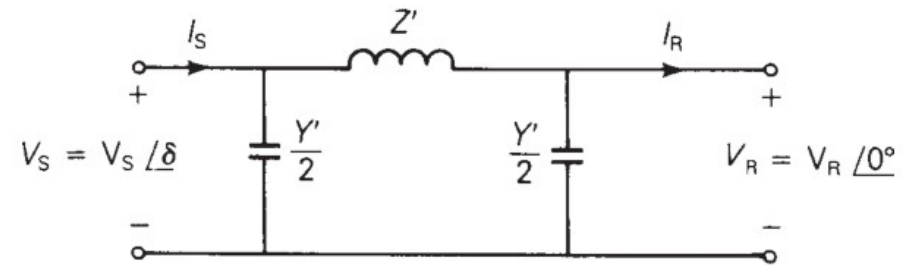
$$Z' = jZ_c \sin(\beta l) = jX' \quad \Omega \quad (5.4.10)$$

$$Z' = (j\omega L l) \left(\frac{\sin(\beta l)}{\beta l} \right) = jX' \quad \Omega \quad (5.4.11)$$

$$\begin{aligned} \frac{Y'}{2} &= \frac{Y}{2} \frac{\tanh(j\beta l / 2)}{j\beta l / 2} = \frac{Y}{2} \frac{\sinh(j\beta l / 2)}{(j\beta l / 2) \cosh(j\beta l / 2)} \\ &= \left(\frac{j\omega C l}{2} \right) \frac{j \sin(\beta l / 2)}{(j\beta l / 2) \cos(\beta l / 2)} = \left(\frac{j\omega C l}{2} \right) \frac{\tan(\beta l / 2)}{\beta l / 2} \\ &= \left(\frac{j\omega C' l}{2} \right) \text{ S} \end{aligned} \quad (5.4.12)$$

FIGURE 5.8

Equivalent π circuit for a lossless line ($\beta \ell$ less than π)



$$Z' = (j\omega L \ell) \left(\frac{\sin \beta \ell}{\beta \ell} \right) = jX' \quad \Omega$$

$$\frac{Y'}{2} = \left(\frac{j\omega C \ell}{2} \right) \frac{\tan(\beta \ell / 2)}{(\beta \ell / 2)} = \frac{j\omega C' \ell}{2} \text{ S}$$

Wavelength

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \quad \text{m} \quad (5.4.15)$$

For a 60-Hz line:

$$\lambda \approx \frac{3 \times 10^8}{60} = 5 \times 10^6 \text{ m} = 5000 \text{ km} = 3100 \text{ mi}$$

Surge impedance loading (SIL)

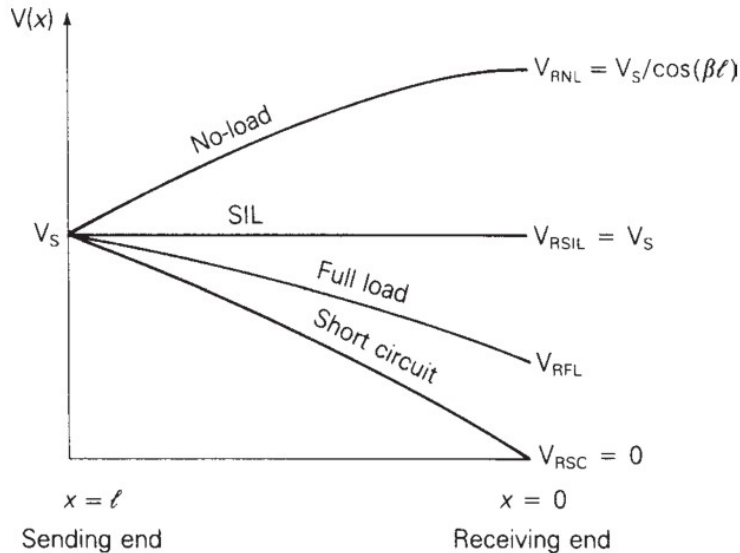
It is the power delivered by a lossless line to a load resistance equal to the surge impedance.

$$\text{SIL} = \frac{V_{\text{rated}}^2}{Z_c} \quad (5.4.21)$$

Voltage profiles

FIGURE 5.10

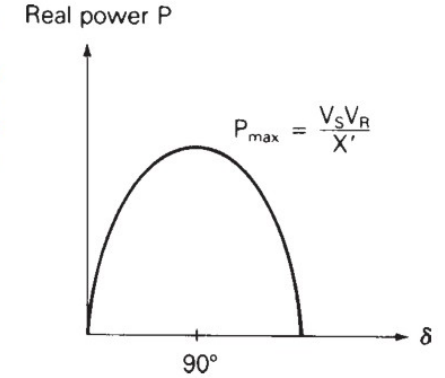
Voltage profiles of an uncompensated lossless line with fixed sending-end voltage for line lengths up to a quarter wavelength



Steady-state stability limit

FIGURE 5.11

Real power delivered by a lossless line versus voltage angle across the line



$$P = P_S = P_R = \text{Re}(S_R) = \frac{V_R V_S}{X'} \sin \delta \quad \text{W} \quad (5.4.26)$$

$$P_{\text{max}} = \frac{V_S V_R}{X'} \quad \text{W} \quad (5.4.27)$$

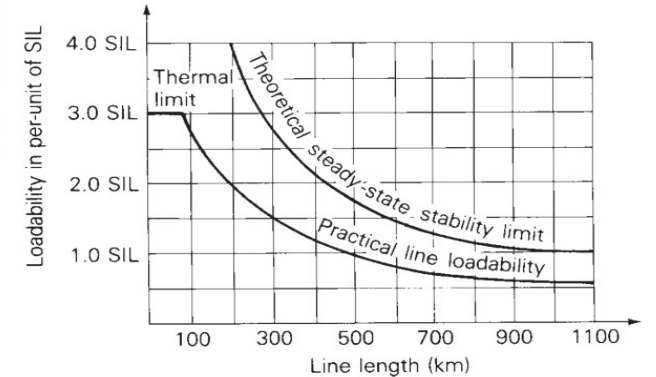
$$P = \frac{V_S V_R \sin \delta}{Z_c \sin \beta l} = \left(\frac{V_S V_R}{Z_c} \right) \frac{\sin \delta}{\sin \left(\frac{2\pi l}{\lambda} \right)} \quad (5.4.28)$$

$$P = \left(\frac{V_S}{V_{\text{rated}}} \right) \left(\frac{V_R}{V_{\text{rated}}} \right) \left(\frac{V_{\text{rated}}^2}{Z_c} \right) \frac{\sin \delta}{\sin \left(\frac{2\pi l}{\lambda} \right)} \\ = V_{\text{S.p.u.}} V_{\text{R.p.u.}} (\text{SIL}) \frac{\sin \delta}{\sin \left(\frac{2\pi l}{\lambda} \right)} \quad \text{W} \quad (5.4.29)$$

$$P_{\text{max}} = \frac{V_{\text{S.p.u.}} V_{\text{R.p.u.}} (\text{SIL})}{\sin \left(\frac{2\pi l}{\lambda} \right)} \quad \text{W} \quad (5.4.30)$$

FIGURE 5.12

Transmission-line loadability curve for 60-Hz overhead lines—no series or shunt compensation



EXAMPLE 5.4 Theoretical steady-state stability limit: long line

Neglecting line losses, find the theoretical steady-state stability limit for the 300-km line in Example 5.2. Assume a $266.1\text{-}\Omega$ surge impedance, a 5000-km wavelength, and $V_S = V_R = 765\text{ kV}$.

From Example 5.2:

A three-phase 765-kV, 60-Hz, 300-km, completely transposed line has the following positive-sequence impedance and admittance:

$$z = 0.0165 + j0.3306 = 0.3310/\underline{87.14^\circ} \quad \Omega/\text{km}$$

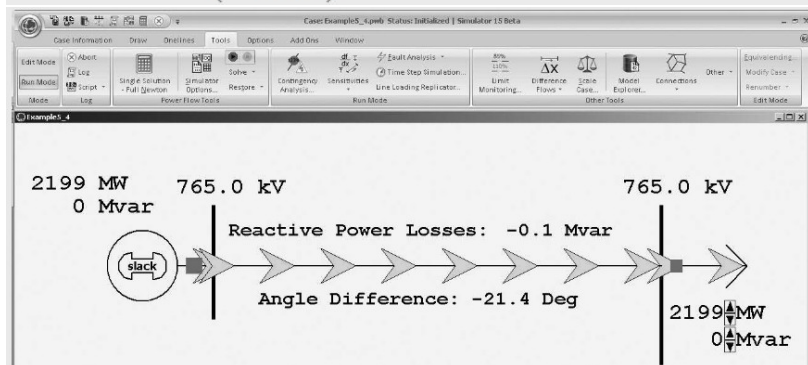
$$y = j4.674 \times 10^{-6} \quad \text{S/km}$$

SOLUTION From (5.4.21),

$$\text{SIL} = \frac{(765)^2}{266.1} = 2199 \text{ MW}$$

From (5.4.30) with $l = 300\text{ km}$ and $\lambda = 5000\text{ km}$,

$$P_{\max} = \frac{(1)(1)(2199)}{\sin\left(\frac{2\pi \times 300}{5000}\right)} = (2.716)(2199) = 5974 \text{ MW}$$



Alternatively, from Figure 5.12, for a 300-km line, the theoretical steady-state stability limit is $(2.72)\text{SIL} = (2.72)(2199) = 5980\text{ MW}$, about the same as the above result (see Figure 5.13).

Open PowerWorld Simulator case Example 5_4 and select **Tools Play** to see an animated view of this example. When the load on a line is equal to the SIL, the voltage profile across the line is flat and the line's net reactive power losses are zero. For loads above the SIL, the line consumes reactive power and the load's voltage magnitude is below the sending-end value. Conversely, for loads below the SIL, the line actually generates reactive power and the load's voltage magnitude is above the sending-end value. Use the load arrow button to vary the load to see the changes in the receiving-end voltage and the line's reactive power consumption. ■

LOSSY LINES

Maximum power flow

Maximum power flow is presented here in terms of the ABCD parameters for lossy lines.

$$A = \cosh(\gamma l) = A/\theta_A$$

$$B = Z' = Z'/\theta_Z$$

$$V_S = V_S/\delta \quad V_R = V_R/\theta^\circ$$

$$P_R = \text{Re}(S_R) = \frac{V_R V_S}{Z'} \cos(\theta_Z - \delta) - \frac{AV_R^2}{Z'} \cos(\theta_Z - \theta_A) \quad (5.5.3)$$

$$Q_R = \text{Im}(S_R) = \frac{V_R V_S}{Z'} \sin(\theta_Z - \delta) - \frac{AV_R^2}{Z'} \sin(\theta_Z - \theta_A) \quad (5.5.4)$$

The theoretical maximum real power delivered (or steady-state stability limit) occurs when $\delta = \theta_Z$ in (5.5.3):

$$P_{R\max} = \frac{V_R V_S}{Z'} - \frac{AV_R^2}{Z'} \cos(\theta_Z - \theta_A) \quad (5.5.6)$$

EXAMPLE 5.5 Theoretical maximum power delivered: long line

The 300-km uncompensated line in Example 5.2 has four 1,272,000-cmil 54/3 ACSR conductors per bundle. The sending-end voltage is held constant at 1.0 per-unit of rated line voltage. Determine the following:

- The practical line loadability. (Assume an approximate receiving-end voltage $V_R = 0.95$ per unit and $\delta = 35^\circ$ maximum angle across the line.)
- The full-load current at 0.986 p.f. leading based on the above practical line loadability
- The exact receiving-end voltage for the full-load current found in part (b)
- Percent voltage regulation for the above full-load current
- Thermal limit of the line, based on the approximate current-carrying capacity given in Table A.4

SOLUTION

- a. From (5.5.3), with $V_S = 765$, $V_R = 0.95 \times 765$ kV, and $\delta = 35^\circ$, using the values of Z' , θ_Z , A , and θ_A from Example 5.5,

$$P_R = \frac{(765)(0.95 \times 765)}{97.0} \cos(87.2^\circ - 35^\circ) - \frac{(0.9313)(0.95 \times 765)^2}{97.0} \cos(87.2^\circ - 0.209^\circ)$$

$$= 3513 - 266 = 3247 \text{ MW}$$

$P_R = 3247$ MW is the practical line loadability, provided the thermal and voltage-drop limits are not exceeded. Alternatively, from Figure 5.12 for a 300-km line, the practical line loadability is $(1.49)\text{SIL} = (1.49)(2199) = 3277$ MW, about the same as the above result.

- b. For the above loading at 0.986 p.f. leading and at 0.95×765 kV, the full-load receiving-end current is

$$I_{RFL} = \frac{P}{\sqrt{3}V_R(\text{p.f.})} = \frac{3247}{(\sqrt{3})(0.95 \times 765)(0.986)} = 2.616 \text{ kA}$$

- c. From (5.1.1) with $I_{RFL} = 2.616/\cos^{-1} 0.986 = 2.616/9.599^\circ$ kA, using the A and B parameters from Example 5.2,

$$V_S = AV_{RFL} + BI_{RFL}$$

$$\frac{765}{\sqrt{3}} \angle \delta = (0.9313/0.209^\circ)(V_{RFL} \angle 0^\circ) + (97.0/87.2^\circ)(2.616/9.599^\circ)$$

$$441.7 \angle \delta = (0.9313V_{RFL} - 30.04) + j(0.0034V_{RFL} + 251.97)$$

Taking the squared magnitude of the above equation,

$$(441.7)^2 = 0.8673V_{RFL}^2 - 54.24V_{RFL} + 64,391$$

Solving,

$$V_{RFL} = 420.7 \text{ kV}_{LN}$$

$$= 420.7\sqrt{3} = 728.7 \text{ kV}_{LL} = 0.953 \text{ per unit}$$

- d. From (5.1.19), the receiving-end no-load voltage is

$$V_{RNL} = \frac{V_S}{A} = \frac{765}{0.9313} = 821.4 \text{ kV}_{LL}$$

And from (5.1.18),

$$\text{percent VR} = \frac{821.4 - 728.7}{728.7} \times 100 = 12.72\%$$

- e. From Table A.4, the approximate current-carrying capacity of four 1,272,000-cmil 54/3 ACSR conductors is $4 \times 1.2 = 4.8$ kA.

Since the voltages $V_S = 1.0$ and $V_{RFL} = 0.953$ per unit satisfy the voltage-drop limit $V_R/V_S \geq 0.95$, the factor that limits line loadability is steady-state stability for this 300-km uncompensated line. The full-load current of 2.616 kA corresponding to loadability is also well below the thermal limit of 4.8 kA. The 12.7% voltage regulation is too high because the no-load voltage is too high. Compensation techniques to reduce no-load voltages are discussed in Section 5.7. ■

LINE LOADABILITY

In practice, power lines are not operated to deliver their theoretical maximum power, which is based on rated terminal voltages and an angular displacement $\delta = 90^\circ$ across the line. Figure 5.12 shows a practical line loadability curve plotted below the theoretical steady-state stability limit. This curve is based on the voltage-drop limit $V_R = V_S \geq 0.95$ and on a maximum angular displacement of 30° to 35° across the line (or about 45° across the line and equivalent system reactances), in order to maintain stability during transient disturbances. The curve is valid for typical overhead 60-Hz lines with no compensation. Note that for short lines less than 80 km long, loadability is limited by the thermal rating of the conductors or by terminal equipment ratings, not by voltage drop or stability considerations.

EXAMPLE 5.6**Practical line loadability and percent voltage regulation: long line**

The 300-km uncompensated line in Example 5.2 has four 1,272,000-cmil 54/3 ACSR conductors per bundle. The sending-end voltage is held constant at 1.0 per-unit of rated line voltage. Determine the following:

- The practical line loadability. (Assume an approximate receiving-end voltage $V_R = 0.95$ per unit and $\delta = 35^\circ$ maximum angle across the line.)
- The full-load current at 0.986 p.f. leading based on the above practical line loadability
- The exact receiving-end voltage for the full-load current found in part (b)

- d. Percent voltage regulation for the above full-load current
- e. Thermal limit of the line, based on the approximate current-carrying capacity given in Table A.4

SOLUTION

- a. From (5.5.3), with $V_S = 765$, $V_R = 0.95 \times 765$ kV, and $\delta = 35^\circ$, using the values of Z' , θ_Z , A , and θ_A from Example 5.5,

$$\begin{aligned} P_R &= \frac{(765)(0.95 \times 765)}{97.0} \cos(87.2^\circ - 35^\circ) \\ &\quad - \frac{(0.9313)(0.95 \times 765)^2}{97.0} \cos(87.2^\circ - 0.209^\circ) \\ &= 3513 - 266 = 3247 \text{ MW} \end{aligned}$$

$P_R = 3247$ MW is the practical line loadability, provided the thermal and voltage-drop limits are not exceeded. Alternatively, from Figure 5.12 for a 300-km line, the practical line loadability is $(1.49)\text{SIL} = (1.49)(2199) = 3277$ MW, about the same as the above result.

- b. For the above loading at 0.986 p.f. leading and at 0.95×765 kV, the full-load receiving-end current is

$$I_{\text{RFL}} = \frac{P}{\sqrt{3}V_R(\text{p.f.})} = \frac{3247}{(\sqrt{3})(0.95 \times 765)(0.986)} = 2.616 \text{ kA}$$

- c. From (5.1.1) with $I_{\text{RFL}} = 2.616/\cos^{-1} 0.986 = 2.616/9.599^\circ$ kA, using the A and B parameters from Example 5.2,

$$V_S = AV_{\text{RFL}} + BI_{\text{RFL}}$$

$$\frac{765}{\sqrt{3}} \angle \delta = (0.9313/0.209^\circ)(V_{\text{RFL}}/0^\circ) + (97.0/87.2^\circ)(2.616/9.599^\circ)$$

$$441.7 \angle \delta = (0.9313V_{\text{RFL}} - 30.04) + j(0.0034V_{\text{RFL}} + 251.97)$$

Taking the squared magnitude of the above equation,

$$(441.7)^2 = 0.8673V_{\text{RFL}}^2 - 54.24V_{\text{RFL}} + 64,391$$

Solving,

$$\begin{aligned} V_{\text{RFL}} &= 420.7 \text{ kV}_{\text{LN}} \\ &= 420.7\sqrt{3} = 728.7 \text{ kV}_{\text{LL}} = 0.953 \text{ per unit} \end{aligned}$$

- d. From (5.1.19), the receiving-end no-load voltage is

$$V_{\text{RNL}} = \frac{V_S}{A} = \frac{765}{0.9313} = 821.4 \text{ kV}_{\text{LL}}$$

And from (5.1.18),

$$\text{percent VR} = \frac{821.4 - 728.7}{728.7} \times 100 = 12.72\%$$

- e. From Table A.4, the approximate current-carrying capacity of four 1,272,000-cmil 54/3 ACSR conductors is $4 \times 1.2 = 4.8$ kA.

Since the voltages $V_S = 1.0$ and $V_{\text{RFL}} = 0.953$ per unit satisfy the voltage-drop limit $V_R/V_S \geq 0.95$, the factor that limits line loadability is steady-state stability for this 300-km uncompensated line. The full-load current of 2.616 kA corresponding to loadability is also well below the thermal limit of 4.8 kA. The 12.7% voltage regulation is too high because the no-load voltage is too high. Compensation techniques to reduce no-load voltages are discussed in Section 5.7. ■

EXAMPLE 5.7

Selection of transmission line voltage and number of lines for power transfer

From a hydroelectric power plant 9000 MW are to be transmitted to a load center located 500 km from the plant. Based on practical line loadability criteria, determine the number of three-phase, 60-Hz lines required to transmit this power, with one line out of service, for the following cases: (a) 345-kV lines with $Z_c = 297 \Omega$; (b) 500-kV lines with $Z_c = 277 \Omega$; (c) 765-kV lines with $Z_c = 266 \Omega$. Assume $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, and $\delta = 35^\circ$. Also assume that the lines are uncompensated and widely separated such that there is negligible mutual coupling between them.

SOLUTION

- a. For 345-kV lines, (5.4.21) yields

$$\text{SIL} = \frac{(345)^2}{297} = 401 \text{ MW}$$

Neglecting losses, from (5.4.29), with $l = 500$ km and $\delta = 35^\circ$,

$$P = \frac{(1.0)(0.95)(401) \sin(35^\circ)}{\sin\left(\frac{2\pi \times 500}{5000}\right)} = (401)(0.927) = 372 \text{ MW/line}$$

Alternatively, the practical line loadability curve in Figure 5.12 can be used to obtain $P = (0.93)\text{SIL}$ for typical 500-km overhead 60-Hz uncompensated lines.

In order to transmit 9000 MW with one line out of service,

$$\#345\text{-kV lines} = \frac{9000 \text{ MW}}{372 \text{ MW/line}} + 1 = 24.2 + 1 \approx 26$$

b. For 500-kV lines,

$$\text{SIL} = \frac{(500)^2}{277} = 903 \text{ MW}$$

$$P = (903)(0.927) = 837 \text{ MW/line}$$

$$\#500\text{-kV lines} = \frac{9000}{837} + 1 = 10.8 + 1 \approx 12$$

c. For 765-kV lines,

$$\text{SIL} = \frac{(765)^2}{266} = 2200 \text{ MW}$$

$$P = (2200)(0.927) = 2039 \text{ MW/line}$$

$$\#765\text{-kV lines} = \frac{9000}{2039} + 1 = 4.4 + 1 \approx 6$$

Increasing the line voltage from 345 to 765 kV, a factor of 2.2, reduces the required number of lines from 26 to 6, a factor of 4.3. ■

EXAMPLE 5.8

Effect of intermediate substations on number of lines required for power transfer

Can five instead of six 765-kV lines transmit the required power in Example 5.7 if there are two intermediate substations that divide each line into three 167-km line sections, and if only one line section is out of service?

SOLUTION The lines are shown in Figure 5.14. For simplicity, we neglect line losses. The equivalent π circuit of one 500-km, 765-kV line has a series reactance, from (5.4.10) and (5.4.15),

$$X' = (266) \sin\left(\frac{2\pi \times 500}{5000}\right) = 156.35 \text{ } \Omega$$

Combining series/parallel reactances in Figure 5.14, the equivalent reactance of five lines with one line section out of service is

$$X_{eq} = \frac{1}{5} \left(\frac{2}{3} X' \right) + \frac{1}{4} \left(\frac{X'}{3} \right) = 0.2167 X' = 33.88 \text{ } \Omega$$

Then, from (5.4.26) with $\delta = 35^\circ$,

$$P = \frac{(765)(765 \times 0.95) \sin(35^\circ)}{33.88} = 9412 \text{ MW}$$

Inclusion of line losses would reduce the above value by 3 or 4% to about 9100 MW. Therefore, the answer is yes. Five 765-kV, 500-km uncompensated lines with two intermediate substations and with one line section out of service will transmit 9000 MW. Intermediate substations are often economical if their costs do not outweigh the reduction in line costs.

This example is modeled in PowerWorld Simulator case Example 5_8 (see Figure 5.15). Each line segment is represented with the lossless line model from Example 5.4 with the π circuit parameters modified to exactly match those for a 167 km distributed line. The pie charts on each line segment show the percentage loading of the line, assuming a rating of 3500 MVA. The solid red squares on the lines represent closed circuit breakers,

FIGURE 5.14

Transmission-line configuration for Example 5.8

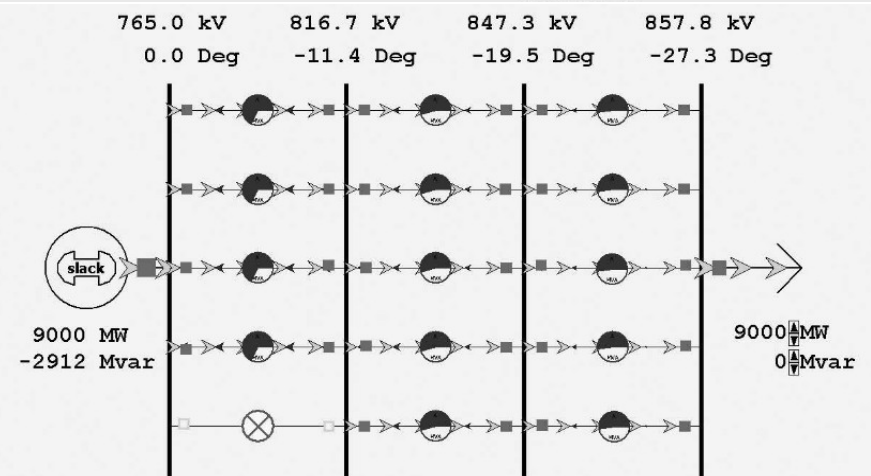
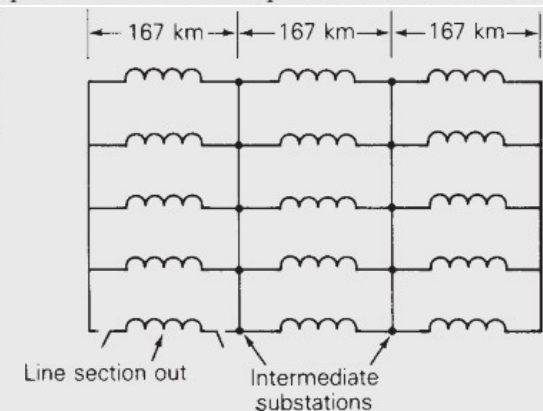


FIGURE 5.15 Screen for Example 5.8

and the green squares correspond to open circuit breakers. Clicking on a circuit breaker toggles its status. The simulation results differ slightly from the simplified analysis done earlier in the example because the simulation includes the charging capacitance of the transmission lines. With all line segments in-service, use the load's arrow to verify that the SIL for this system is 11,000 MW, five times that of the single circuit line in Example 5.4. ■

REACTIVE COMPENSATION TECHNIQUES

Inductors and capacitors are used on medium-length and long transmission lines to increase line loadability and to maintain voltages near rated values.

SHUNT COMPENSATION

A. Shunt reactors (inductors)

Commonly installed at selected points along EHV lines from each phase to neutral,

1. They absorb reactive power and reduce overvoltages during light load conditions and reduce transient overvoltages due to switching and lightning surges.
2. They can reduce line loadability if not removed under full-load conditions.

B. Shunt capacitors

Sometimes used to deliver reactive power and increase transmission voltages during heavy load conditions.

C. Static var compensators

They are thyristor-switched reactors in parallel with capacitors,

1. Can absorb reactive power during light loads and deliver reactive power during heavy loads.
2. Through automatic control of the thyristor switches, voltage fluctuations are minimized and line loadability is increased.

D. Synchronous condensers

Synchronous motors with no mechanical load. They can also control their reactive power output, although more slowly than static var compensators.

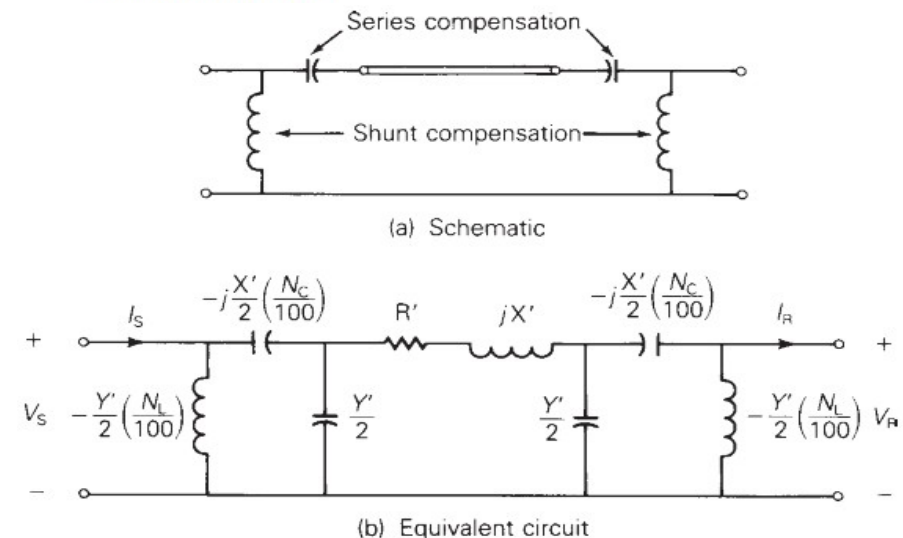
SERIES COMPENSATION

Capacitor banks installed in series with each phase conductor at selected points along a line.

1. Series capacitors are sometimes used on long lines to increase line loadability.
2. Their effect is to reduce the net series impedance of the line in series with the capacitor banks, thereby reducing line-voltage drops and increasing the steady-state stability limit.
3. Disadvantages
 - a. Automatic protection devices must be installed to by-pass high currents during faults and to reinsert the capacitor banks after fault clearing.
 - b. Can excite low-frequency oscillations, a phenomenon called subsynchronous resonance, which may damage turbine-generator shafts.
4. Studies have shown, however, that series capacitive compensation can increase the loadability of long lines at only a fraction of the cost of new transmission.

FIGURE 5.16

Compensated transmission-line section



N_C is the amount of series capacitive compensation expressed in percent of the positive-sequence line impedance and N_L is the amount of shunt reactive compensation in percent of the positive-sequence line admittance.

EXAMPLE 5.9**Shunt reactive compensation to improve transmission-line voltage regulation**

Identical shunt reactors (inductors) are connected from each phase conductor to neutral at both ends of the 300-km line in Example 5.2 during light load conditions, providing 75% compensation. The reactors are removed during heavy load conditions. Full load is 1.90 kA at unity p.f. and at 730 kV. Assuming that the sending-end voltage is constant, determine the following:

- Percent voltage regulation of the uncompensated line
- The equivalent shunt admittance and series impedance of the compensated line
- Percent voltage regulation of the compensated line

SOLUTION

- a. From (5.1.1) with $I_{RFL} = 1.9/0^\circ$ kA, using the A and B parameters from Example 5.2,

$$\begin{aligned} V_S &= AV_{RFL} + BI_{RFL} \\ &= (0.9313/0.209^\circ) \left(\frac{730}{\sqrt{3}}/0^\circ \right) + (97.0/87.2^\circ)(1.9/0^\circ) \\ &= 392.5/0.209^\circ + 184.3/87.2^\circ \\ &= 401.5 + j185.5 \\ &= 442.3/24.8^\circ \text{ kV}_{LN} \end{aligned}$$

$$V_S = 442.3\sqrt{3} = 766.0 \text{ kV}_{LL}$$

The no-load receiving-end voltage is, from (5.1.19),

$$V_{RNL} = \frac{766.0}{0.9313} = 822.6 \text{ kV}_{LL}$$

and the percent voltage regulation for the uncompensated line is, from (5.1.18),

$$\text{percent VR} = \frac{822.6 - 730}{730} \times 100 = 12.68\%$$

- b. From Example 5.3, the shunt admittance of the equivalent π circuit without compensation is

$$\begin{aligned} Y' &= 2(3.7 \times 10^{-7} + j7.094 \times 10^{-4}) \\ &= 7.4 \times 10^{-7} + j14.188 \times 10^{-4} \text{ S} \end{aligned}$$

With 75% shunt compensation, the equivalent shunt admittance is

$$\begin{aligned} Y_{eq} &= 7.4 \times 10^{-7} + j14.188 \times 10^{-4} \left(1 - \frac{75}{100}\right) \\ &= 3.547 \times 10^{-4}/89.88^\circ \text{ S} \end{aligned}$$

Since there is no series compensation, the equivalent series impedance is the same as without compensation:

$$Z_{eq} = Z' = 97.0/87.2^\circ \Omega$$

- c. The equivalent A parameter for the compensated line is

$$\begin{aligned} A_{eq} &= 1 + \frac{Y_{eq}Z_{eq}}{2} \\ &= 1 + \frac{(3.547 \times 10^{-4}/89.88^\circ)(97.0/87.2^\circ)}{2} \\ &= 1 + 0.0172/177.1^\circ \\ &= 0.9828/0.05^\circ \text{ per unit} \end{aligned}$$

Then, from (5.1.19),

$$V_{RNL} = \frac{766}{0.9828} = 779.4 \text{ kV}_{LL}$$

Since the shunt reactors are removed during heavy load conditions, $V_{RFL} = 730$ kV is the same as without compensation. Therefore

$$\text{percent VR} = \frac{779.4 - 730}{730} \times 100 = 6.77\%$$

The use of shunt reactors at light loads improves the voltage regulation from 12.68% to 6.77% for this line. ■

EXAMPLE 5.10**Series capacitive compensation to increase transmission-line loadability**

Identical series capacitors are installed in each phase at both ends of the line in Example 5.2, providing 30% compensation. Determine the theoretical maximum power that this compensated line can deliver and compare with that of the uncompensated line. Assume $V_S = V_R = 765$ kV.

SOLUTION From Example 5.3, the equivalent series reactance without compensation is

$$X' = 97.0 \sin 87.2^\circ = 96.88 \Omega$$

Based on 30% series compensation, half at each end of the line, the impedance of each series capacitor is

$$Z_{\text{cap}} = -jX_{\text{cap}} = -j\left(\frac{1}{2}\right)(0.30)(96.88) = -j14.53 \quad \Omega$$

From Figure 5.4, the $ABCD$ matrix of this series impedance is

$$\begin{bmatrix} 1 & -j14.53 \\ 0 & 1 \end{bmatrix}$$

As also shown in Figure 5.4, the equivalent $ABCD$ matrix of networks in series is obtained by multiplying the $ABCD$ matrices of the individual networks. For this example there are three networks: the series capacitors at the sending end, the line, and the series capacitors at the receiving end. Therefore the equivalent $ABCD$ matrix of the compensated line is, using the $ABCD$ parameters, from Example 5.2,

$$\begin{bmatrix} 1 & -j14.53 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.9313/\underline{0.209^\circ} & 97.0/\underline{87.2^\circ} \\ 1.37 \times 10^{-3}/\underline{90.06^\circ} & 0.9313/\underline{0.209^\circ} \end{bmatrix} \begin{bmatrix} 1 & -j14.53 \\ 0 & 1 \end{bmatrix}$$

After performing these matrix multiplications, we obtain

$$\begin{bmatrix} A_{\text{eq}} & B_{\text{eq}} \\ C_{\text{eq}} & D_{\text{eq}} \end{bmatrix} = \begin{bmatrix} 0.9512/\underline{0.205^\circ} & 69.70/\underline{86.02^\circ} \\ 1.37 \times 10^{-3}/\underline{90.06^\circ} & 0.9512/\underline{0.205^\circ} \end{bmatrix}$$

Therefore

$$A_{\text{eq}} = 0.9512 \quad \text{per unit} \quad \theta_{A_{\text{eq}}} = 0.205^\circ$$

$$B_{\text{eq}} = Z'_{\text{eq}} = 69.70 \quad \Omega \quad \theta_{Z_{\text{eq}}} = 86.02^\circ$$

From (5.5.6) with $V_S = V_R = 765 \text{ kV}$,

$$\begin{aligned} P_{\text{Rmax}} &= \frac{(765)^2}{69.70} - \frac{(0.9512)(765)^2}{69.70} \cos(86.02^\circ - 0.205^\circ) \\ &= 8396 - 583 = 7813 \quad \text{MW} \end{aligned}$$

which is 36.2% larger than the value of 5738 MW found in Example 5.5 without compensation. We note that the practical line loadability of this series compensated line is also about 35% larger than the value of 3247 MW found in Example 5.6 without compensation.

This example is modeled in PowerWorld Simulator case Example 5_10 (see Figure 5.17). When opened, both of the series capacitors are bypassed (i.e., they are modeled as short circuits) meaning this case is initially identical to the Example 5.4 case. Click on the blue “Bypassed” field to place each of the series capacitors into the circuit. This decreases the angle across the line, resulting in more net power transfer.

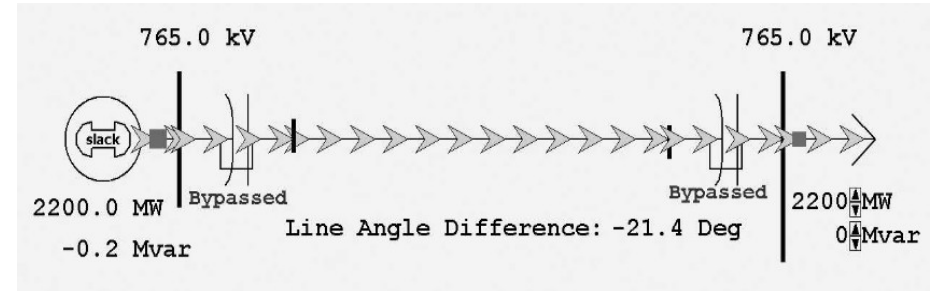


FIGURE 5.17 Screen for Example 5.10

TRANSIENT STABILITY STUDIES

Simplifying assumptions:

1. Only balanced three-phase systems and balanced disturbances are considered. Therefore, only positive-sequence networks are employed.
2. Deviations of machine frequencies from synchronous frequency (60 Hz) are small, and offset currents and harmonics are neglected. Therefore, the network of transmission lines, transformers, and impedance loads is essentially in steady-state; and voltages, currents, and powers can be computed from algebraic power-flow equations.

SYNCHRONOUS MACHINE ROTOR DYNAMICS: THE SWING EQUATION

Rotor motion of a three-phase synchronous generator and its prime mover

By Newton's second law,

$$J\alpha_m(t) = T_m(t) - T_e(t) = T_a(t) \quad (11.1.1)$$

where J = total moment of inertia of the rotating masses, kgm^2

α_m = rotor angular acceleration, rad/s^2

T_m = mechanical torque supplied by the prime mover minus the retarding torque due to mechanical losses, Nm

T_e = electrical torque that accounts for the total three-phase electrical power output of the generator, plus electrical losses, Nm

T_a = net accelerating torque, Nm

Also, the rotor angular acceleration is given by

$$\alpha_m(t) = \frac{d\omega_m(t)}{dt} = \frac{d^2\theta_m(t)}{dt^2} \quad (11.1.2)$$

$$\omega_m(t) = \frac{d\theta_m(t)}{dt} \quad (11.1.3)$$

where ω_m = rotor angular velocity, rad/s

θ_m = rotor angular position with respect to a stationary axis, rad

T_m vs T_e

T_m and T_e are positive for generator operation.

When $T_m = T_e$ (steady-state), $T_a = 0$, $\alpha_m = 0$ and ω_m = constant synchronous speed.

When $T_m > T_e$, $T_a = +$, $\alpha_m > 0$ and ω_m increases.

When $T_m < T_e$, ω_m decreases.

Let

ω_{msyn} = synchronous angular velocity of the rotor, rad/s

δ_m = rotor angular position with respect to a synchronously rotating reference, rad

Then

$$\theta_m(t) = \omega_{msyn}t + \delta_m(t) \quad (11.1.4)$$

Using (11.1.2) and (11.1.4), (11.1.1) becomes

$$J \frac{d^2\theta_m(t)}{dt^2} = J \frac{d^2\delta_m(t)}{dt^2} = T_m(t) - T_e(t) = T_a(t) \quad (11.1.5)$$

In terms of power and p.u values,

$$\begin{aligned} \frac{J\omega_m(t)}{S_{rated}} \frac{d^2\delta_m(t)}{dt^2} &= \frac{\omega_m(t)T_m(t) - \omega_m(t)T_e(t)}{S_{rated}} \\ &= \frac{p_m(t) - p_e(t)}{S_{rated}} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \end{aligned} \quad (11.1.6)$$

where $p_{mp.u.}$ = mechanical power supplied by the prime mover minus mechanical losses, per unit

$p_{ep.u.}$ = electrical power output of the generator plus electrical losses, per unit

Normalized Inertia Constant, H :

By definition,

$$\begin{aligned} H &= \frac{\text{stored kinetic energy at synchronous speed}}{\text{generator voltampere rating}} \\ &= \frac{\frac{1}{2} J \omega_{msyn}^2}{S_{rated}} \quad \text{joules/VA or per unit-seconds} \end{aligned} \quad (11.1.7)$$

The H constant has the advantage that it falls within a fairly narrow range, normally between 1 and 10 p.u.-s, whereas J varies widely, depending on generating unit size and type. Solving (11.1.7) for J and using in (11.1.6),

$$2H \frac{\omega_m(t)}{\omega_{msyn}^2} \frac{d^2\delta_m(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \quad (11.1.8)$$

Defining per-unit rotor angular velocity,

$$\omega_{p.u.}(t) = \frac{\omega_m(t)}{\omega_{msyn}} \quad (11.1.9)$$

Equation (11.1.8) becomes

$$\frac{2H}{\omega_{msyn}} \omega_{p.u.}(t) \frac{d^2\delta_m(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \quad (11.1.10)$$

For a synchronous generator with P poles, the electrical angular acceleration α , electrical radian frequency ω , and power angle δ are

$$\alpha(t) = \frac{P}{2} \alpha_m(t) \quad (11.1.11)$$

$$\omega(t) = \frac{P}{2} \omega_m(t) \quad (11.1.12)$$

$$\delta(t) = \frac{P}{2} \delta_m(t) \quad (11.1.13)$$

Similarly, the synchronous electrical radian frequency is

$$\omega_{syn} = \frac{P}{2} \omega_{msyn} \quad (11.1.14)$$

The per-unit electrical frequency is

$$\omega_{p.u.}(t) = \frac{\omega(t)}{\omega_{syn}} = \frac{\frac{P}{2} \omega(t)}{\frac{P}{2} \omega_{syn}} = \frac{\omega_m(t)}{\omega_{msyn}} \quad (11.1.15)$$

Therefore, using (11.1.13–11.1.15), (11.1.10) can be written as

$$\frac{2H}{\omega_{syn}} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) = p_{ap.u.}(t) \quad (11.1.16)$$

Damping coefficient, D

Using D, a term that represents a damping torque anytime the generator deviates from its synchronous speed, with its value proportional to the speed deviation

$$\begin{aligned} & 2H/\omega_{syn} \omega_{p.u.}(t) (d^2\delta(t)/(dt^2)) \\ &= p_{mp.u.}(t) - p_{ep.u.}(t) - D/\omega_{syn} (d\delta(t)/(dt)) \\ &= p_{ap.u.}(t) \end{aligned} \quad (11.1.17)$$

where D is either zero or a relatively small positive number with typical values between 0 and 2. The units of D are per unit power divided by per unit speed deviation.

Equation (11.1.17), called **the per-unit swing equation**, is the fundamental equation that determines rotor dynamics in transient stability studies.

The per-unit swing equation as two first-order differential equations

Diffrentiating (11.1.4), and then using (11.1.3) and (11.1.12)–(11.1.14), we obtain

$$\frac{d\delta(t)}{dt} = \omega(t) - \omega_{syn} \quad (11.1.18)$$

Using (11.1.18) in (11.1.17),

$$\frac{2H}{\omega_{syn}} \omega_{p.u.}(t) \frac{d\omega(t)}{dt} = p_{mp.u.}(t) - p_{ep.u.}(t) - D/\omega_{syn} \frac{d\delta(t)}{dt} = p_{ap.u.}(t) \quad (11.1.19)$$

EXAMPLE 11.1 Generator per-unit swing equation and power angle during a short circuit

A three-phase, 60-Hz, 500-MVA, 15-kV, 32-pole hydroelectric generating unit has an H constant of 2.0 p.u.-s and $D = 0$. (a) Determine ω_{syn} and ω_{msyn} . (b) Give the per-unit swing equation for this unit. (c) The unit is initially operating at $p_{mp.u.} = p_{ep.u.} = 1.0$, $\omega = \omega_{syn}$, and $\delta = 10^\circ$ when a three-phase-to-ground bolted short circuit at the generator terminals causes $p_{ep.u.}$ to drop to zero for $t \geq 0$. Determine the power angle 3 cycles after the short circuit commences. Assume $p_{mp.u.}$ remains constant at 1.0 per unit. Also assume $\omega_{p.u.}(t) = 1.0$ in the swing equation.

SOLUTION

a. For a 60-Hz generator,

$$\omega_{\text{syn}} = 2\pi 60 = 377 \text{ rad/s}$$

and, from (11.1.14), with $P = 32$ poles,

$$\omega_{\text{msyn}} = \frac{2}{P} \omega_{\text{syn}} = \left(\frac{2}{32}\right) 377 = 23.56 \text{ rad/s}$$

b. From (11.1.16), with $H = 2.0$ p.u.-s,

$$\frac{4}{2\pi 60} \omega_{\text{p.u.}}(t) \frac{d^2 \delta(t)}{dt^2} = p_{\text{mp.u.}}(t) - p_{\text{ep.u.}}(t)$$

c. The initial power angle is

$$\delta(0) = 10^\circ = 0.1745 \text{ radian}$$

Also, from (11.1.17), at $t = 0$,

$$\frac{d\delta(0)}{dt} = 0$$

Using $p_{\text{mp.u.}}(t) = 1.0$, $p_{\text{ep.u.}} = 0$, and $\omega_{\text{p.u.}}(t) = 1.0$, the swing equation from (b) is

$$\left(\frac{4}{2\pi 60}\right) \frac{d^2 \delta(t)}{dt^2} = 1.0 \quad t \geq 0$$

Integrating twice and using the above initial conditions,

$$\begin{aligned} \frac{d\delta(t)}{dt} &= \left(\frac{2\pi 60}{4}\right) t + 0 \\ \delta(t) &= \left(\frac{2\pi 60}{8}\right) t^2 + 0.1745 \end{aligned}$$

At $t = 3 \text{ cycles} = \frac{3 \text{ cycles}}{60 \text{ cycles/second}} = 0.05 \text{ second}$,

$$\begin{aligned} \delta(0.05) &= \left(\frac{2\pi 60}{8}\right) (0.05)^2 + 0.1745 \\ &= 0.2923 \text{ radian} = 16.75^\circ \end{aligned}$$

EXAMPLE 11.2 Equivalent swing equation: two generating units

A power plant has two three-phase, 60-Hz generating units with the following ratings:

Unit 1: 500 MVA, 15 kV, 0.85 power factor, 32 poles, $H_1 = 2.0$ p.u.-s, $D = 0$

Unit 2: 300 MVA, 15 kV, 0.90 power factor, 16 poles, $H_2 = 2.5$ p.u.-s, $D = 0$

- (a) Give the per-unit swing equation of each unit on a 100-MVA system base.
 (b) If the units are assumed to “swing together,” that is, $\delta_1(t) = \delta_2(t)$, combine the two swing equations into one equivalent swing equation.

SOLUTION

a. If the per-unit powers on the right-hand side of the swing equation are converted to the system base, then the H constant on the left-hand side must also be converted. That is,

$$H_{\text{new}} = H_{\text{old}} \frac{S_{\text{old}}}{S_{\text{new}}} \text{ per unit}$$

Converting H_1 from its 500-MVA rating to the 100-MVA system base,

$$H_{1\text{new}} = H_{1\text{old}} \frac{S_{\text{old}}}{S_{\text{new}}} = (2.0) \left(\frac{500}{100}\right) = 10 \text{ p.u.-s}$$

Similarly, converting H_2 ,

$$H_{2\text{new}} = (2.5) \left(\frac{300}{100}\right) = 7.5 \text{ p.u.-s}$$

The per-unit swing equations on the system base are then

$$\begin{aligned} \frac{2H_{1\text{new}}}{\omega_{\text{syn}}} \omega_{1\text{p.u.}}(t) \frac{d^2 \delta_1(t)}{dt^2} &= \frac{20.0}{2\pi 60} \omega_{1\text{p.u.}}(t) \frac{d^2 \delta_1(t)}{dt^2} \\ &= p_{m1\text{p.u.}}(t) - p_{e1\text{p.u.}}(t) \\ \frac{2H_{2\text{new}}}{\omega_{\text{syn}}} \omega_{2\text{p.u.}}(t) \frac{d^2 \delta_2(t)}{dt^2} &= \frac{15.0}{2\pi 60} \omega_{2\text{p.u.}}(t) \frac{d^2 \delta_2(t)}{dt^2} = p_{m2\text{p.u.}}(t) - p_{e2\text{p.u.}}(t) \end{aligned}$$

b. Letting:

$$\delta(t) = \delta_1(t) = \delta_2(t)$$

$$\omega_{\text{p.u.}}(t) = \omega_{1\text{p.u.}}(t) = \omega_{2\text{p.u.}}(t)$$

$$p_{mp.u.}(t) = p_{m1p.u.}(t) + p_{m2p.u.}(t)$$

$$p_{ep.u.}(t) = p_{e1p.u.}(t) + p_{e2p.u.}(t)$$

and adding the above swing equations

$$\begin{aligned} \frac{2(H_{1new} + H_{2new})}{\omega_{syn}} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2} \\ = \frac{35.0}{2\pi 60} \omega_{p.u.}(t) \frac{d^2\delta(t)}{dt^2} = p_{mp.u.}(t) - p_{ep.u.}(t) \end{aligned}$$

When transient stability studies involving large-scale power systems with many generating units are performed with a digital computer, computation time can be reduced by combining the swing equations of those units that swing together. Such units, which are called *coherent machines*, usually are connected to the same bus or are electrically close, and they are usually remote from network disturbances under study. ■

SIMPLIFIED SYNCHRONOUS MACHINE MODEL AND SYSTEM EQUIVALENTS

Figure 11.2 shows a simplified model of a synchronous machine, called the **classical model**, that can be used in transient stability studies. This model is based on the following assumptions:

1. The machine is operating under balanced three-phase positive-sequence conditions.
2. Machine excitation is constant.
3. Machine losses, saturation, and saliency are neglected.

In transient stability programs, more detailed models can be used to represent exciters, losses, saturation, and saliency. However, the simplified model reduces model complexity while maintaining reasonable accuracy in stability calculations.

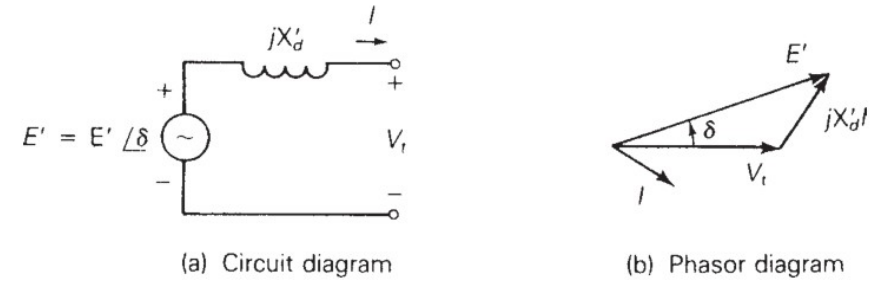


FIGURE 11.2
Simplified synchronous machine model for transient stability studies

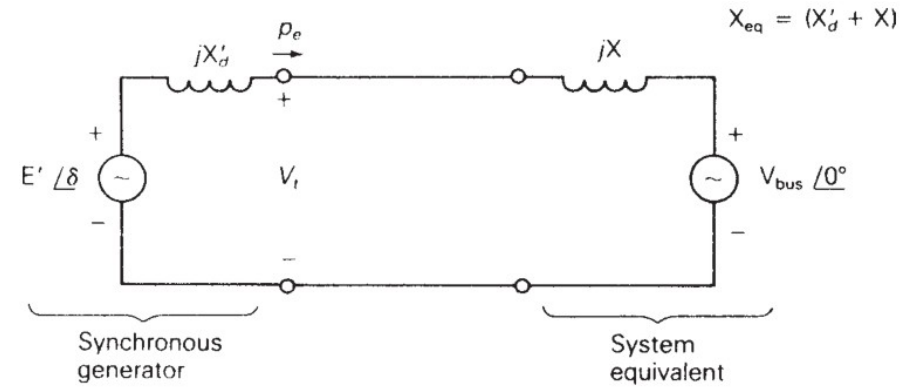


FIGURE 11.3
Synchronous generator connected to a system equivalent

The **phase angle δ** of the internal machine voltage is the machine power angle with respect to the infinite bus.

Real power delivered by the synchronous generator to the infinite bus:

$$p_e = \frac{E' V_{bus}}{X_{eq}} \sin \delta \quad (11.2.1)$$

where $X_{eq} = (X'_d + X)$ = the equivalent reactance between the machine internal voltage and infinite bus.

During transient disturbances both E' and V_{bus} are considered constant in (11.2.1). Thus p_e is a sinusoidal function of the machine power angle δ .

EXAMPLE 11.3 Generator internal voltage and real power output versus power angle

Figure 11.4 shows a single-line diagram of a three-phase, 60-Hz synchronous generator, connected through a transformer and parallel transmission lines to an infinite bus. All reactances are given in per-unit on a common system base. If the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging, determine (a) the internal voltage of the generator and (b) the equation for the electrical power delivered by the generator versus its power angle δ .

SOLUTION

a. The equivalent circuit is shown in Figure 11.5, from which the equivalent reactance between the machine internal voltage and infinite bus is

$$\begin{aligned} X_{eq} &= X'_d + X_{TR} + X_{12} \parallel (X_{13} + X_{23}) \\ &= 0.30 + 0.10 + 0.20 \parallel (0.10 + 0.20) \\ &= 0.520 \text{ per unit} \end{aligned}$$

The current into the infinite bus is

$$\begin{aligned} I &= \frac{P}{V_{bus}(\text{p.f.})} \angle -\cos^{-1}(\text{p.f.}) = \frac{(1.0)}{(1.0)(0.95)} \angle -\cos^{-1} 0.95 \\ &= 1.05263 \angle -18.195^\circ \text{ per unit} \end{aligned}$$

FIGURE 11.4 Single-line diagram for Example 11.3

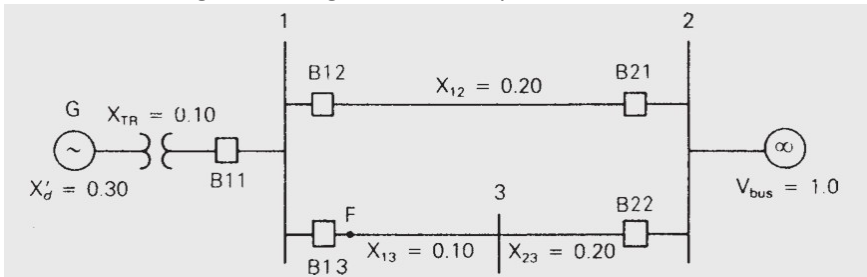
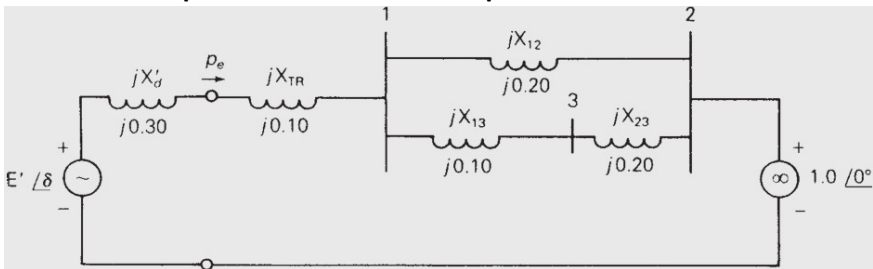


FIGURE 11.5 Equivalent circuit for Example 11.3



and the machine internal voltage is

$$\begin{aligned} E' &= E' / \delta = V_{bus} + jX_{eq}I \\ &= 1.0 \angle 0^\circ + (j0.520)(1.05263 \angle -18.195^\circ) \\ &= 1.0 \angle 0^\circ + 0.54737 \angle 71.805^\circ \\ &= 1.1709 + j0.5200 \\ &= 1.2812 \angle 23.946^\circ \text{ per unit} \end{aligned}$$

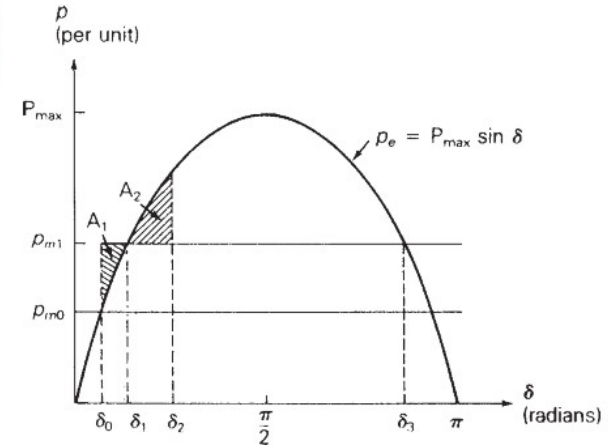
b. From (11.2.1),

$$p_e = \frac{(1.2812)(1.0)}{0.520} \sin \delta = 2.4638 \sin \delta \text{ per unit}$$

THE EQUAL-AREA CRITERION

FIGURE 11.6

p_e and p_m versus δ



Operating point oscillation

Suppose the unit is initially operating in steady-state at $p_e = p_m = p_{m0}$ and $\delta = \delta_0$, when a step change in p_m from p_{m0} to p_{m1} occurs at $t = 0$. Due to rotor inertia, the rotor position cannot change instantaneously. That is, $\delta_m(0^+) = \delta_m(0^-)$; therefore, $\delta(0^+) = \delta(0^-) = \delta_0$ and $p_e(0^+) = p_e(0^-)$. Since $p_m(0^+) = p_{m1}$ is greater than $p_e(0^+)$, the acceleration power $p_a(0^+)$ is positive and, from (11.1.16), $(d^2\delta)/(dt^2)(0^+)$ is positive. The rotor accelerates and δ increases. When δ reaches δ_1 , $p_e = p_{m1}$ and $(d^2\delta)/(dt^2)$ becomes zero. However, $d\delta/dt$ is still positive and δ continues to increase, overshooting its final steady-state operating point. When δ is greater than δ_1 , p_m is less than p_e , p_a is negative, and the rotor decelerates. Eventually, δ reaches a maximum value

δ_2 and then swings back toward δ_1 . Using (11.1.16), which has no damping, δ would continually oscillate around δ_1 . However, damping due to mechanical and electrical losses causes δ to stabilize at its final steady-state operating point δ_1 . Note that if the power angle exceeded δ_3 , then p_m would exceed p_e and the rotor would accelerate again, causing a further increase in δ and loss of stability.

Determining stability and maximum power angle

1. Use numerical integration techniques using a digital computer to solve the nonlinear swing equation via
2. Use the equal-area criterion method.

The equal-area criterion: $A_1 = A_2$

In Figure 11.6, p_m is greater than p_e during the interval $\delta_0 < \delta < \delta_1$, and the rotor is accelerating. The shaded area A_1 between the p_m and p_e curves is called the accelerating area. During the interval $\delta_1 < \delta < \delta_2$, p_m is less than p_e , the rotor is decelerating, and the shaded area A_2 is the decelerating area. At both the initial value $\delta = \delta_0$ and the maximum value $\delta = \delta_2$, $d\delta/dt = 0$.

Deriving the equal-area criterion: $A_1 = A_2$

To derive the equal-area criterion for one machine connected to an infinite bus, assume $\omega_{p.u.}(t) = 1$ in (11.1.16), giving

$$\frac{2H}{\omega_{syn}} \frac{d^2\delta}{dt^2} = p_{mp.u.} - p_{ep.u.} \quad (11.3.1)$$

Multiplying by $d\delta/dt$ and using

$$\frac{d}{dt} \left[\frac{d\delta}{dt} \right]^2 = 2 \left(\frac{d\delta}{dt} \right) \left(\frac{d^2\delta}{dt^2} \right)$$

(11.3.1) becomes

$$\frac{2H}{\omega_{syn}} \left(\frac{d^2\delta}{dt^2} \right) \left(\frac{d\delta}{dt} \right) = \frac{H}{\omega_{syn}} \frac{d}{dt} \left[\frac{d\delta}{dt} \right]^2 = (p_{mp.u.} - p_{ep.u.}) \frac{d\delta}{dt} \quad (11.3.2)$$

Multiplying (11.3.2) by dt and integrating from δ_0 to δ ,

$$\frac{H}{\omega_{syn}} \int_{\delta_0}^{\delta} d \left[\frac{d\delta}{dt} \right]^2 = \int_{\delta_0}^{\delta} (p_{mp.u.} - p_{ep.u.}) d\delta$$

or

$$\frac{H}{\omega_{syn}} \left[\frac{d\delta}{dt} \right]^2 \Big|_{\delta_0}^{\delta} = \int_{\delta_0}^{\delta} (p_{mp.u.} - p_{ep.u.}) d\delta \quad (11.3.3)$$

The above integration begins at δ_0 where $d\delta/dt = 0$, and continues to an arbitrary δ . When δ reaches its maximum value, denoted δ_2 , $d\delta/dt$ again equals zero. Therefore, the left-hand side of (11.3.3) equals zero for $\delta = \delta_2$ and

$$\int_{\delta_0}^{\delta_2} (p_{mp.u.} - p_{ep.u.}) d\delta = 0 \quad (11.3.4)$$

Separating this integral into positive (accelerating) and negative (decelerating) areas, we arrive at the equal-area criterion

$$\int_{\delta_0}^{\delta_1} (p_{mp.u.} - p_{ep.u.}) d\delta + \int_{\delta_1}^{\delta_2} (p_{mp.u.} - p_{ep.u.}) d\delta = 0$$

or

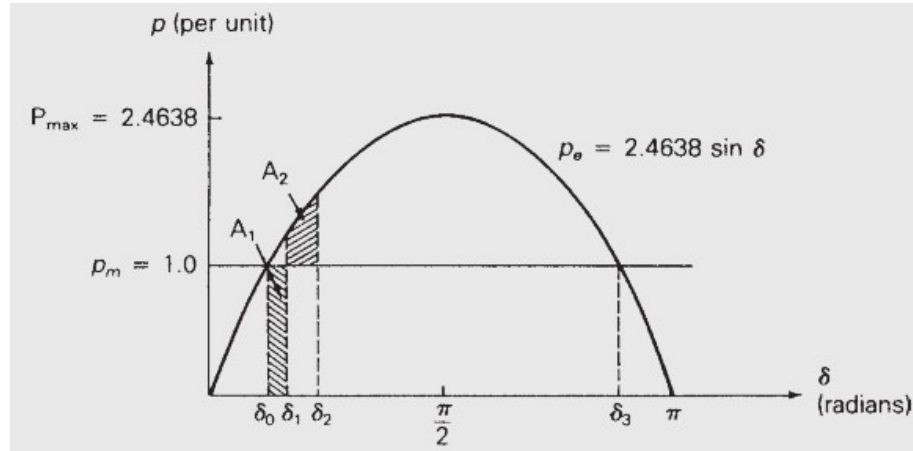
$$\int_{\delta_0}^{\delta_1} \underbrace{(p_{mp.u.} - p_{ep.u.}) d\delta}_{A_1} = \int_{\delta_1}^{\delta_2} \underbrace{(p_{ep.u.} - p_{mp.u.}) d\delta}_{A_2} \quad (11.3.5)$$

In practice, sudden changes in mechanical power usually do not occur, since the time constants associated with prime mover dynamics are on the order of seconds. However, stability phenomena similar to that described above can also occur from sudden changes in electrical power, due to system faults and line switching. The following three examples are illustrative.

EXAMPLE 11.4 Equal-area criterion: transient stability during a three-phase fault

The synchronous generator shown in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3, when a temporary three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 1, shown as point F in Figure 11.4. Three cycles later the fault extinguishes by itself. Due to a relay misoperation, all circuit breakers remain closed. Determine whether stability is or is not maintained and determine the maximum power angle. The inertia constant of the generating unit is 3.0 per unit-seconds on the system base. Assume p_m remains constant throughout the disturbance. Also assume $\omega_{p.u.}(t) = 1.0$ in the swing equation.

FIGURE 11.7 p - δ plot for Example 11.4



SOLUTION Plots of p_e and p_m versus δ are shown in Figure 11.7. From Example 11.3 the initial operating point is $p_e(0^-) = p_m = 1.0$ per unit and $\delta(0^+) = \delta(0^-) = \delta_0 = 23.95^\circ = 0.4179$ radian. At $t = 0$, when the short circuit occurs, p_e instantaneously drops to zero and remains at zero during the fault since power cannot be transferred past faulted bus 1. From (11.1.16), with $\omega_{p.u.}(t) = 1.0$,

$$\frac{2H}{\omega_{syn}} \frac{d^2\delta(t)}{dt^2} = p_{mp.u.} \quad 0 \leq t \leq 0.05 \text{ s}$$

Integrating twice with initial condition $\delta(0) = \delta_0$ and $\frac{d\delta(0)}{dt} = 0$,

$$\begin{aligned} \frac{d\delta(t)}{dt} &= \frac{\omega_{syn} p_{mp.u.}}{2H} t + 0 \\ \delta(t) &= \frac{\omega_{syn} p_{mp.u.}}{4H} t^2 + \delta_0 \end{aligned}$$

At $t = 3$ cycles = 0.05 second,

$$\begin{aligned} \delta_1 = \delta(0.05 \text{ s}) &= \frac{2\pi 60}{12} (0.05)^2 + 0.4179 \\ &= 0.4964 \text{ radian} = 28.44^\circ \end{aligned}$$

The accelerating area A_1 , shaded in Figure 11.7, is

$$A_1 = \int_{\delta_0}^{\delta_1} p_m d\delta = \int_{\delta_0}^{\delta_1} 1.0 d\delta = (\delta_1 - \delta_0) = 0.4964 - 0.4179 = 0.0785$$

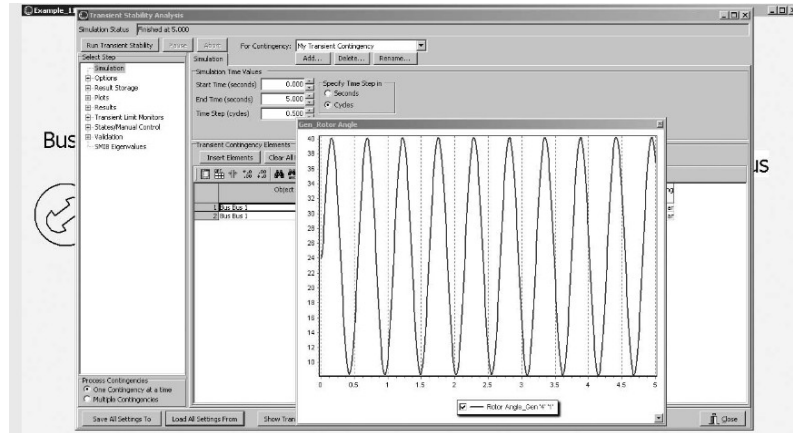


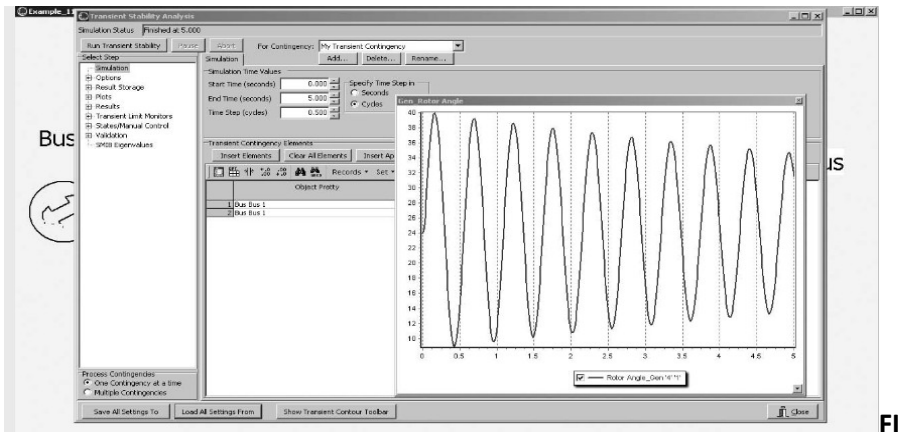
FIGURE 11.8 Variation in $\delta(t)$ without Damping

At $t = 0.05$ s the fault extinguishes and p_e instantaneously increases from zero to the sinusoidal curve in Figure 11.7. δ continues to increase until the decelerating area A_2 equals A_1 . That is,

$$\begin{aligned} A_2 &= \int_{\delta_1}^{\delta_2} (p_{max} \sin \delta - p_m) d\delta \\ &= \int_{0.4964}^{\delta_2} (2.4638 \sin \delta - 1.0) d\delta = A_1 = 0.0785 \end{aligned}$$

Integrating,

$$\begin{aligned} 2.4638 [\cos(0.4964) - \cos \delta_2] - (\delta_2 - 0.4964) &= 0.0785 \\ 2.4638 \cos \delta_2 + \delta_2 &= 2.5843 \end{aligned}$$



FI

FIGURE 11.9 Variation in $\delta(t)$ with Damping

The above nonlinear algebraic equation can be solved iteratively to obtain

$$\delta_2 = 0.7003 \text{ radian} = 40.12^\circ$$

Since the maximum angle δ_2 does not exceed $\delta_3 = (180^\circ - \delta_0) = 156.05^\circ$, stability is maintained. In steady-state, the generator returns to its initial operating point $p_{\text{ess}} = p_m = 1.0$ per unit and $\delta_{\text{ss}} = \delta_0 = 23.95^\circ$.

Note that as the fault duration increases, the risk of instability also increases. The *critical clearing time*, denoted t_{cr} , is the longest fault duration allowable for stability.

To see this case modeled in PowerWorld Simulator, open case Example 11_4 (see Figure 11.8). Then select **Add-Ons, Transient Stability**, which displays the Transient Stability Analysis Form. Notice that in the Transient Stability Contingency Elements list, a fault is applied to bus 1 at $t = 0$ s and

cleared at $t = 0.05$ s (three cycles later). To see the time variation in the generator angle (modeled at bus 4 in PowerWorld Simulator), click the **Run Transient Stability** button. When the simulation is finished, a graph showing this angle automatically appears, as shown in the figure. More detailed results are also available by clicking on **Results** in the list on the left side of the form. To rerun the example with a different fault duration, overwrite the Time (second) field in the Transient Contingency Elements list, and then again click the **Run Transient Stability** button.

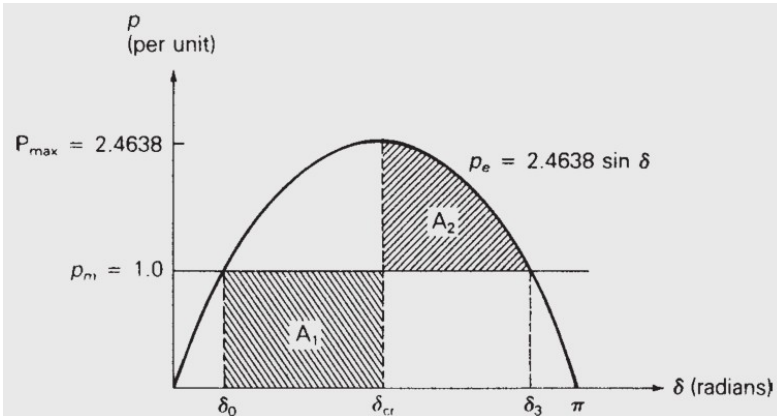
Notice that because this system is modeled without damping (i.e., $D = 0$), the angle oscillations do not damp out with time. To extend the example, right-click on the Bus 4 generator on the one-line diagram and select **Generator Information Dialog**. Then click on the **Stability, Machine Models** tab to see the parameters associated with the GENCLS model (i.e., a classical model—a more detailed machine model is introduced in Section 11.6). Change the “ D ” field to 1.0, select **OK** to close the dialog, and then rerun the transient stability case. The results are as shown in Figure 11.9. While the inclusion of damping did not significantly alter the maximum for $\delta(t)$, the magnitude of the angle oscillations is now decreasing with time. For convenience this modified example is contained in PowerWorld Simulator case Example 11.4b. ■

EXAMPLE 11.5 Equal-area criterion: critical clearing time for a temporary three-phase fault

Assuming the temporary short circuit in Example 11.4 lasts longer than 3 cycles, calculate the critical clearing time.

SOLUTION The p – δ plot is shown in Figure 11.10. At the critical clearing angle, denoted δ_{cr} , the fault is extinguished. The power angle then increases to a maximum value $\delta_3 = 180^\circ - \delta_0 = 156.05^\circ = 2.7236$ radians, which gives

FIGURE 11.10 p – δ plot for Example 11.5



the maximum decelerating area. Equating the accelerating and decelerating areas,

$$A_1 = \int_{\delta_0}^{\delta_{cr}} p_m d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{max} \sin \delta - p_m) d\delta$$

$$\int_{0.4179}^{\delta_{cr}} 1.0 d\delta = \int_{\delta_{cr}}^{2.7236} (2.4638 \sin \delta - 1.0) d\delta$$

Solving for δ_{cr} ,

$$(\delta_{cr} - 0.4179) = 2.4638[\cos \delta_{cr} - \cos(2.7236)] - (2.7236 - \delta_{cr})$$

$$2.4638 \cos \delta_{cr} = +0.05402$$

$$\delta_{cr} = 1.5489 \text{ radians} = 88.74^\circ$$

From the solution to the swing equation given in Example 11.4,

$$\delta(t) = \frac{\omega_{syn} P_{mp.u.}}{4H} t^2 + \delta_0$$

Solving

$$t = \sqrt{\frac{4H}{\omega_{syn} P_{mp.u.}} (\delta(t) - \delta_0)}$$

Using $\delta(t_{cr}) = \delta_{cr} = 1.5489$ and $\delta_0 = 0.4179$ radian,

$$t_{cr} = \sqrt{\frac{12}{(2\pi 60)(1.0)} (1.5489 - 0.4179)}$$

$$= 0.1897 \text{ s} = 11.38 \text{ cycles}$$

If the fault is cleared before $t = t_{cr} = 11.38$ cycles, stability is maintained. Otherwise, the generator goes out of synchronism with the infinite bus; that is, stability is lost.

To see a time-domain simulation of this case, open Example 11.5 in PowerWorld Simulator (see Figure 11.11). Again select **Add-Ons, Transient Stability** to view the Transient Stability Analysis Form. In order to better visualize the results on a PowerWorld one-line diagram, there is an option to transfer the transient stability results to the one-line every n timesteps. To access this option, select the **Options** page from the list on the left side of the display, then the **General** tab, then check the **Transfer Results to Power Flow after Interval Check** field. With this option checked, click on the **Run Transient Stability**, which will run the case with a critical clearing time of 0.1895 seconds. The plot is set to dynamically update as well. The final results are shown in Figure 11.11. Because the one-line is reanimated every n timesteps (4 in this case), a potential downside to this option is it takes longer to run. Uncheck the option to restore full solution speed.

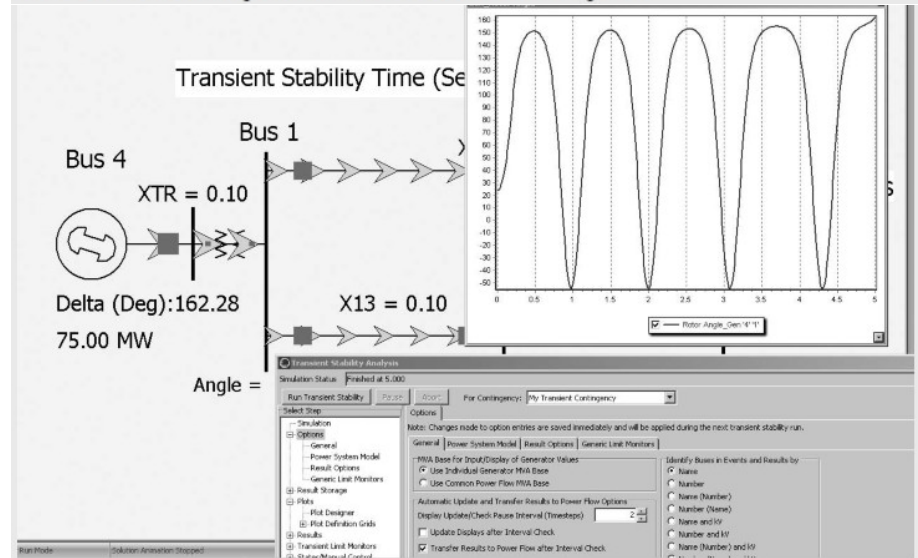


FIGURE 11.11 Variation in $\delta(t)$ for Example 11.5

EXAMPLE 11.6 Equal-area criterion: critical clearing angle for a cleared three-phase fault

The synchronous generator in Figure 11.4 is initially operating in the steady-state condition given in Example 11.3 when a permanent three-phase-to-ground bolted short circuit occurs on line 1–3 at bus 3. The fault is cleared by opening the circuit breakers at the ends of line 1–3 and line 2–3. These circuit breakers then remain open. Calculate the critical clearing angle. As in previous examples, $H = 3.0$ p.u.-s, $p_m = 1.0$ per unit and $\omega_{p.u.} = 1.0$ in the swing equation.

SOLUTION From Example 11.3, the equation for the prefault electrical power, denoted p_{e1} here, is $p_{e1} = 2.4638 \sin \delta$ per unit. The faulted network is

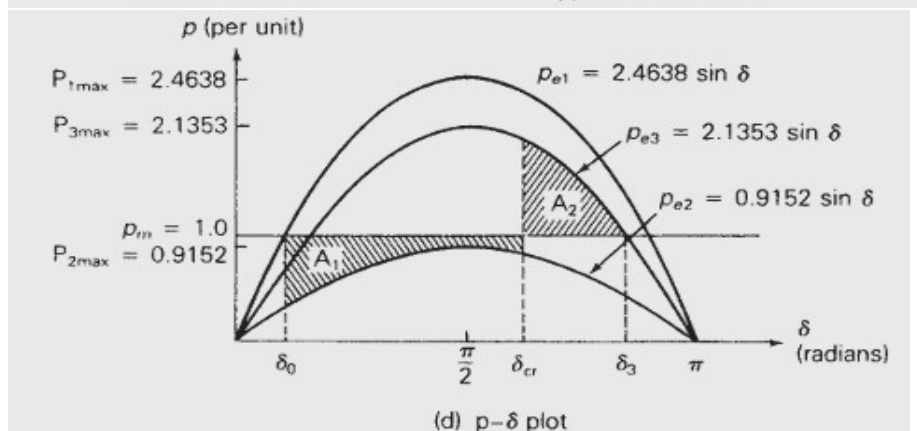
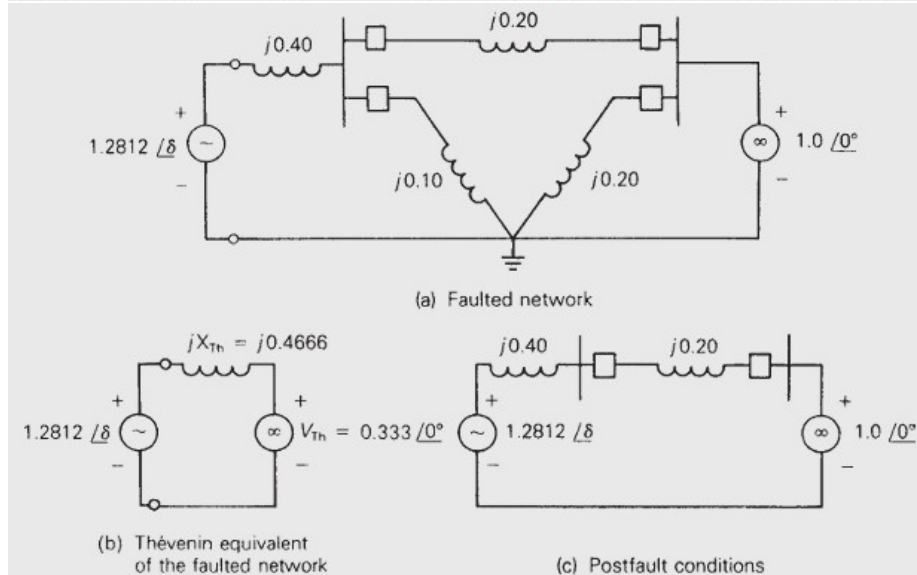


FIGURE 11.12 Example 11.6

shown in Figure 11.12(a), and the Thévenin equivalent of the faulted network, as viewed from the generator internal voltage source, is shown in Figure 11.12(b). The Thévenin reactance is

$$X_{Th} = 0.40 + 0.20 \parallel 0.10 = 0.46666 \text{ per unit}$$

and the Thévenin voltage source is

$$\begin{aligned} V_{Th} &= 1.0 \angle 0^\circ \left[\frac{X_{13}}{X_{13} + X_{12}} \right] = 1.0 \angle 0^\circ \frac{0.10}{0.30} \\ &= 0.33333 \angle 0^\circ \text{ per unit} \end{aligned}$$

From Figure 11.12(b), the equation for the electrical power delivered by the generator to the infinite bus during the fault, denoted p_{e2} , is

$$p_{e2} = \frac{E' V_{Th}}{X_{Th}} \sin \delta = \frac{(1.2812)(0.3333)}{0.46666} \sin \delta = 0.9152 \sin \delta \text{ per unit}$$

The postfault network is shown in Figure 11.12(c), where circuit breakers have opened and removed lines 1–3 and 2–3. From this figure, the postfault electrical power delivered, denoted p_{e3} , is

$$p_{e3} = \frac{(1.2812)(1.0)}{0.60} \sin \delta = 2.1353 \sin \delta \text{ per unit}$$

The p - δ curves as well as the accelerating area A_1 and decelerating area A_2 corresponding to critical clearing are shown in Figure 11.12(d). Equating A_1 and A_2 ,

$$\begin{aligned} A_1 &= \int_{\delta_0}^{\delta_{cr}} (p_m - P_{2max} \sin \delta) d\delta = A_2 = \int_{\delta_{cr}}^{\delta_3} (P_{3max} \sin \delta - p_m) d\delta \\ \int_{0.4179}^{\delta_{cr}} (1.0 - 0.9152 \sin \delta) d\delta &= \int_{\delta_{cr}}^{2.6542} (2.1353 \sin \delta - 1.0) d\delta \end{aligned}$$

Solving for δ_{cr} ,

$$\begin{aligned} (\delta_{cr} - 0.4179) + 0.9152(\cos \delta_{cr} - \cos 0.4179) \\ = 2.1353(\cos \delta_{cr} - \cos 2.6542) - (2.6542 - \delta_{cr}) \end{aligned}$$

$$-1.2201 \cos \delta_{cr} = 0.4868$$

$$\delta_{cr} = 1.9812 \text{ radians} = 111.5^\circ$$

If the fault is cleared before $\delta = \delta_{\text{cr}} = 111.5^\circ$, stability is maintained. Otherwise, stability is lost. To see this case in PowerWorld Simulator open case Example 11_6. ■